MORLAB – Model Order Reduction LABoratory

Peter Benner[1], Steffen W. R. Werner[1]

[1]Max Planck Institute for Dynamics of Complex Technical Systems, Computational Methods in Systems and Control Theory, Sandtorstr. 1, 39106 Magdeburg, Germany

MORLAB for linear dynamical systems

The **MORLAB**, **M**odel **O**rder **R**eduction **LAB**oratory, toolbox is an open-source collection of routines for model reduction of dense linear time-invariant systems compatible with MathWorks MATLAB and GNU Octave.

The implementation is based on solutions of matrix equations, iterative and spectral projection methods, like the matrix sign and disk functions.

Features

- Open source and free: GNU AGPLv3+
- Fast: implementation uses spectral projection methods
- Unified framework: one interface for all MOR techniques
- Configurable: customization of routines via option structs
- Modular: each subroutine can be directly used
- Dependencies: MATLAB ($\geq$ 2012b) or Octave ($\geq$ 4.0)
- Portable: no binary extensions required

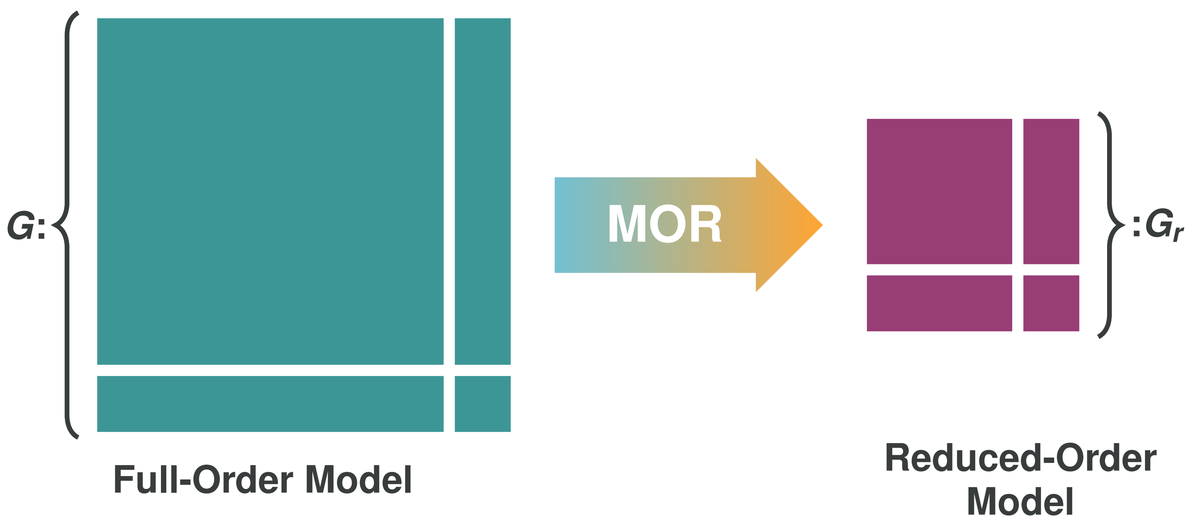
Model reduction problem

For a dynamical system

$$G: \begin{cases} 0 = f(x(t), Dx(t), \ldots, D^k x(t), u(t)), \\ y(t) = h(x(t), Dx(t), \ldots, D^k x(t), u(t)), \end{cases}$$

with states $x(t) \in \mathbb{R}^n$, inputs $u(t) \in \mathbb{R}^p$ and outputs $y(t) \in \mathbb{R}^p$, the goal is to find a surrogate model G_r with a much smaller number of internal states $x_r(t) \in \mathbb{R}^r$, $r \ll n$, that is approximating the input-output mapping

$$\|y - y_r\| \leq \mathrm{tol} \cdot \|u\|.$$



Supported system classes

Standard systems

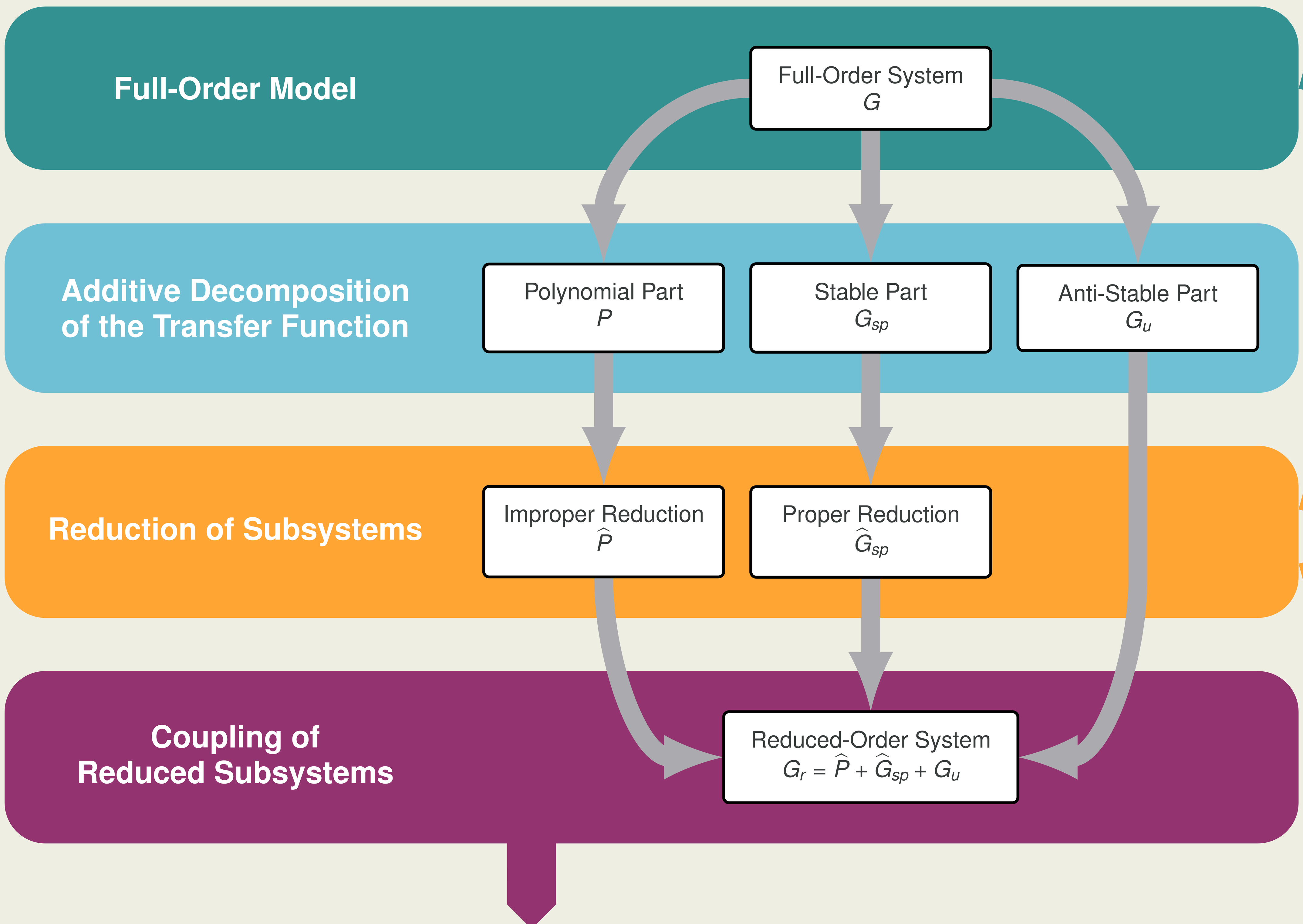
$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x_{k+1} = Ax_k + Bu_k,$$
$$y(t) = Cx(t) + Du(t) \qquad y_k = Cx_k + Du_k$$

Generalized state-space / descriptor systems

$$E\dot{x}(t) = Ax(t) + Bu(t), \qquad Ex_{k+1} = Ax_k + Bu_k,$$
$$y(t) = Cx(t) + Du(t) \qquad y_k = Cx_k + Du_k$$

Second-order systems

$$M\ddot{x}(t) = -Kx(t) - E\dot{x}(t) + B_u u(t),$$
$$y(t) = C_p x(t) + C_v \dot{x}(t) + Du(t)$$



Implemented MOR methods

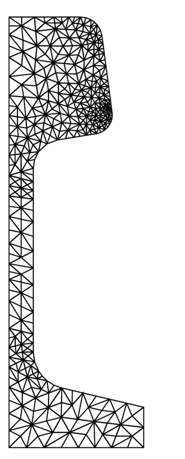
- Modal truncation
- Hankel-norm approximation
- Balanced truncation
- Frequency-limited balanced truncation
- Time-limited balanced truncation
- Balanced stochastic truncation
- Bounded-real balanced truncation
- Positive-real balanced truncation
- $\mathcal{H}_\infty$ balanced truncation
- Linear-quadratic-Gaussian balanced truncation
- Second-order balanced truncation
- Second-order frequency-limited balanced truncation
- Second-order time-limited balanced truncation

Algebraic matrix equation solvers

- Sylvester equations: $AXE + FXB + C = 0$
- Bernoulli equations: $A^T XE + E^T XA - E^T XBB^T XE = 0$
- Lyapunov equations: $AXE^T + EXA^T + BB^T = 0$
  $AXA^T - EXE^T + BB^T = 0$
- Riccati equations: $A^T XE + E^T XA + C^T C + \alpha E^T XBB^T XE = 0$
  $A^T XA - E^T XE + C^T C + \alpha A^T XB(I_m - \alpha BXB^T)B^T XA = 0$

Benchmark example

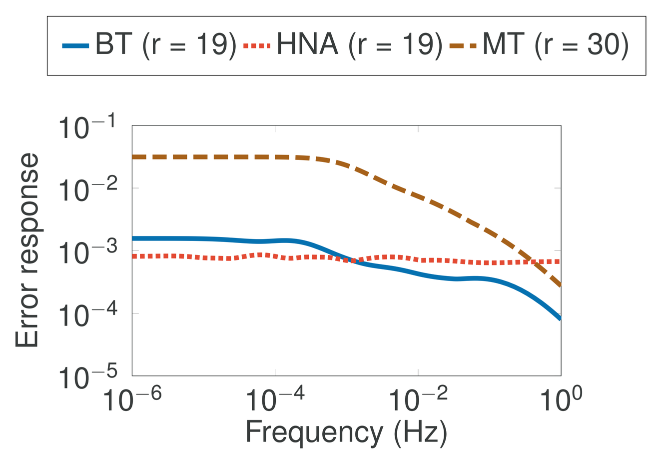
Semi-discretized heat equation on a steel profile[2]



The semi-discretization of the unsteady heat equation leads to a system of linear ordinary differential equations:

- stable system with
- $n = 5177$, $m = 7$, $p = 6$.

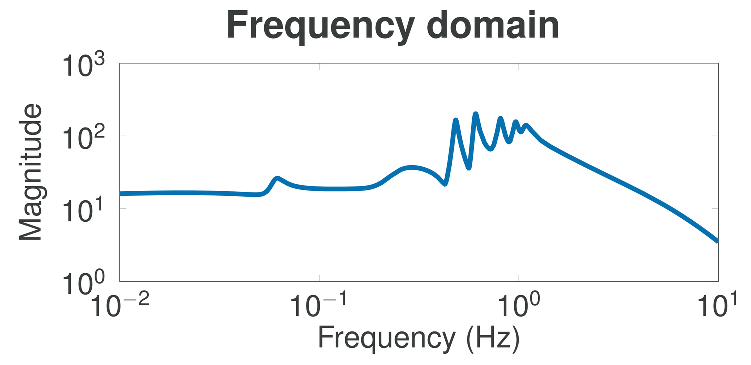
For the model reduction, the balanced truncation (BT), Hankel-norm approximation (HNA) and modal truncation (MT) have been computed.

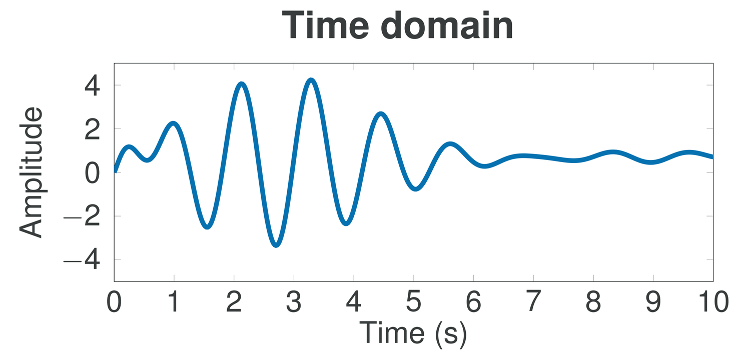


[2]Oberwolfach Benchmark Collection, Steel profile. hosted at MORwiki – Model Order Reduction Wiki, 2005.

System evaluation and visualization features

The toolbox provides a set of evaluation and visualization tools for the supported system classes:





Plots: Sigma, Frobenius, Bode magnitude

Simulators: SS11, SS21, SS22, implicit Euler

Power systems examples. hosted at MORwiki – Model Order Reduction Wiki, 2005.

README

[1] P. Benner, *A MATLAB repository for model reduction based on spectral projection*, in 2006 IEEE Conference on Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control, Oct. 2006, pp. 19–24.

[2] P. Benner and S. W. R. Werner, *MORLAB - Modellreduktion in MATLAB*, in Tagungsband GMA-FA 1.30 'Modellierung, Identifikation und Simulation in der Automatisierungstechnik' und GMA-FA 1.40 'Theoretische Verfahren der Regelungstechnik', Workshop in Anif, Salzburg, 18.-22.09.2017, T. Meurer and F. Woittennek, eds., 2017, pp. 508–517.

[3] ——, *Balancing related model reduction with the MORLAB toolbox*, Proc. Appl. Math. Mech., 18 (2018), p. e201800083.

[4] ——, *Model reduction of descriptor systems with the MORLAB toolbox*, IFAC-PapersOnLine 9th Vienna International Conference on Mathematical Modelling MATHMOD 2018, Vienna, Austria, 21–23 February 2018, 51 (2018), pp. 547–552.

[5] ——, *MORLAB – Model Order Reduction LABoratory (version 5.0)*, Aug. 2018. see also: `http://www.mpi-magdeburg.mpg.de/projects/morlab`. doi:10.5281/zenodo.3332716.